Quality Competition in a Duopoly: Stochastic Qualities and Heterogeneous Firms

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**MOTIVATION**

- Existing literature on quality differentiation in duopolies mainly focus on symmetric firms and quality choice under certainty.
- By focusing on identical firms, the previous models fail to explain why certain firms choose to produce higher quality than others in a vertically differentiated product market.
- The uncertain nature of quality improving R&D investments implies that, realistically, firms will not be able to choose their qualities with 100% certainty. Thus, it is of interest to analyze the isolated effect of firm uncertainty when making quality investments.

**BASELINE MODEL**

- We adopt the baseline model for our analysis from Tirole (1988) and Belleflamme and Pietz (2010).
- Assume there are two firms that each produce a good with reservation value \( r \) and can vary on a continuum of relative quality levels: \( s_i \in [s^-, s^+] \).
- Consumer valuations of quality are uniformly distributed on \( \theta \in [\theta^-, \theta^+] \); the utility function:
  \[
  U(\theta, s_i, p_i) = r + \theta s_i - p_i.
  \]
- We analyze the following game:
  - Stage 1: Firms set their qualities simultaneously.
  - Stage 2: Firms observe each others qualities and set their prices simultaneously.
  - Stage 3: Consumers purchase exactly one product between the two firms.
- **Proposition 3.2** (Tirole 1988). There are two pure strategy Nash equilibria in the quality stage of the above game. Each involve one firm choosing the lowest relative quality while the other firm produces the highest relative quality product.
- **Mixed Strategies**: The two classes of mixed strategy equilibria each consists of one firm mixing along the entire interval of relative qualities while the other firm mixes exclusively between \( s^- \) and \( s^+ \).

**HETEROGENEOUS FIRMS**

We first consider two firms that face asymmetric costs to quality improvement. In this model, each firm incurs an \( \alpha_i s_i^2 \) fixed cost to improve quality.

**Propositions 4.5 and 4.7**. If \( s^+ \) is sufficiently high and \( \alpha_i \gg \alpha_j \), then the unique pure strategy Nash equilibrium in the quality stage will involve firm \( i \) choosing to produce the lowest relative quality while firm \( j \) chooses to produce a higher quality product.

**Remarks**: Figure 1 shows the profits of firm \( j \) as a function of its quality given fixed qualities of firm \( i \). Note that firm \( j \) will choose to produce a higher quality product even if firm \( i \) chooses to produce a good above the worst relative quality.

**UNCERTAIN QUALITY CHOICE**

Our results imply that differing firm structures offer an explanation to why certain firms produce higher qualities than others.

**UNCERTAIN QUALITY INVESTMENT**

We extend the uncertain quality choice model: to receive a distribution \( U[0, b_i] \), each firm has to incur an investment cost of \( ab_i^2 \). We find that \( \alpha \) and \( B \) uniquely determines the class of pure strategy Nash equilibria in the quality stage (Proposition 6.10).

**Corollary 5.4.2**. The unique dominant strategy (Nash) equilibrium in the quality stage involves both firms choosing the highest possible target quality.

**CONCLUSIONS**

- Sufficient differences in returns to quality investment leads to a unique pure strategy Nash equilibrium in the quality stage.
- Stochastic quality realization is an incentive for quality investment.
- Uncertain quality softens price competition and prevents firms from direct Bertrand competition.
- Firms no longer need to intentionally differentiate their choices in the quality stage.

**REFERENCES**

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