Quality Competition in a Duopoly: Stochastic Qualities and Heterogeneous Firms

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MOTIVATION

- Existing literature on quality differentiation in duopolies mainly focus on symmetric firms and quality choice under certainty.
- By focusing on identical firms, the previous models fail to explain *why* certain firms choose to produce higher quality than others in a vertically differentiated product market.
- The uncertain nature of quality improving R&D investments implies that, realistically, firms will not be able to choose their qualities with 100% certainty. Thus, it is of interested to analyze the *isolated* effect of firm uncertainty when making quality investments.

BASELINE MODEL

- We adopt the baseline model for our analysis from Tirole (1988) and Belleflamme and Pietz (2010).
- Assume there are two firms that each produce a good with reservation value r and can vary on a continuum of relative quality levels: $s_i \in [s^-, s^+]$.
- Consumer valuations of quality are uniformly distributed on $\theta \in [\theta^-, \theta^+]$; the utility function:

$$U(\theta, s_i, p_i) = r + \theta s_i - p_i.$$

- We analyze the following game:
  - ➢ Stage 1: Firms set their qualities simultaneously.
  - ➢ Stage 2: Firms observe each others qualities and set their prices simultaneously.
  - ➢ Stage 3: Consumers purchase exactly one product between the two firms.
- **Proposition 3.2** (Tirole 1988)**.** There are two pure strategy Nash equilibria in the quality stage of the above game. Each involve one firm choosing the lowest relative quality while the other firm produces the highest relative quality product.
- **Mixed Strategies:** The two classes of mixed strategy equilibria each consists of one firm mixing along the entire interval of relative qualities while the other firm mixes exclusively between s^- and s^+.

HETEROGENEOUS FIRMS

We first consider two firms that face asymmetric costs to quality improvement. In this model, each firm incurs an $\alpha_i s_i^2$ *fixed cost* to improve quality.

Propositions 4.5 and 4.7. If s^+ is sufficiently high and $\alpha_i \gg \alpha_j$, then the unique pure strategy Nash equilibrium in the quality stage will involve firm i choosing to produce the lowest relative quality while firm j chooses to produce a higher quality product.

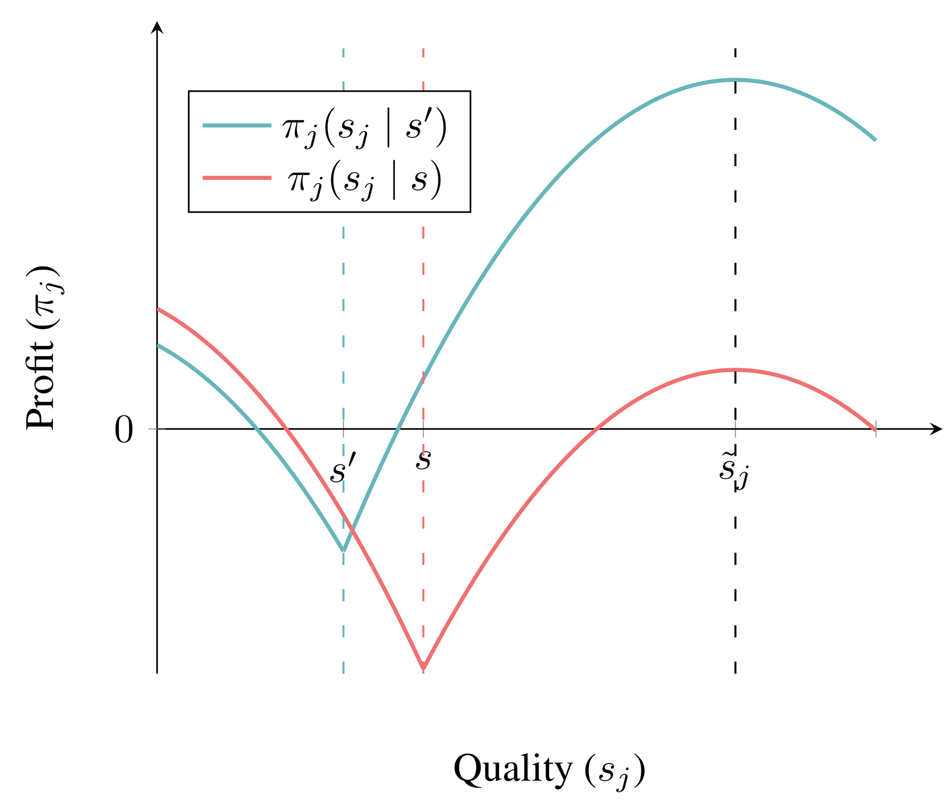

Figure 1. Profits with respect to Quality Investment

Remarks: *Figure 1* shows the profits of firm j as a function of its quality *given* fixed qualities of firm i. Note that firm j will choose to produce a higher quality product even if firm i chooses to produce a good above the worst relative quality.

Our results imply that differing firm structures offer an explanation to *why* certain firms produce higher qualities than others.

UNCERTAIN QUALITY CHOICE

In this model, each firm can only *choose* a $b_i \in [0, B]$ that acts as its target quality. The actual quality is the realization of the random variable $\xi_i \sim \mathcal{U}[0, b_i]$.

Corollary 5.4.2. The unique dominant strategy (Nash) equilibrium in the quality stage involves *both* firms choosing the highest possible target quality.

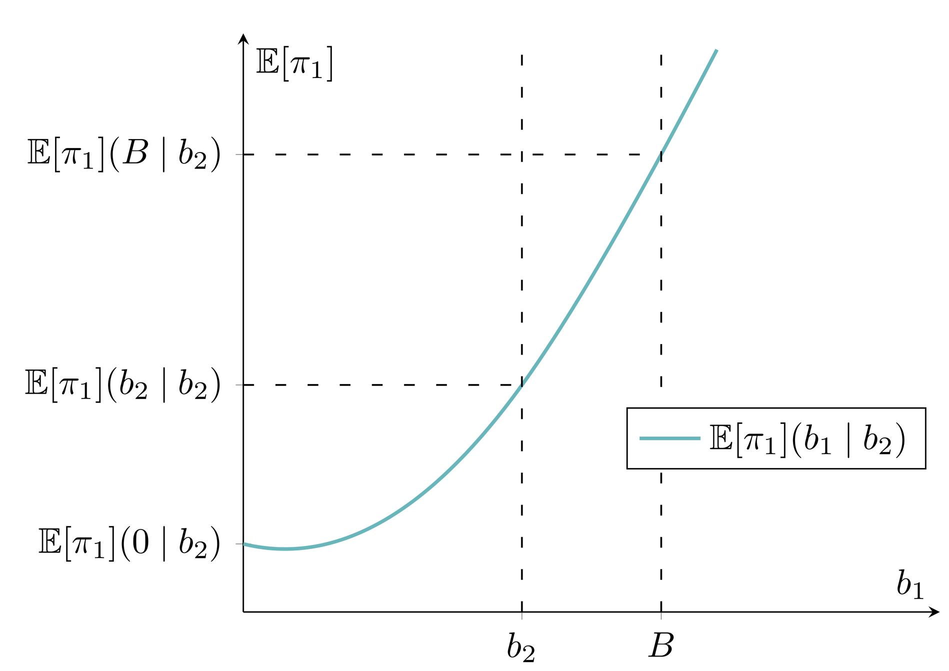

Figure 2. Expected Profits with respect to Target Quality

Remarks: *Figure 2* graphs the expected profits of firm 1 as a function of its target quality. Although it's initially decreasing, the eventual increase yields $b_1 = B$ as the profit maximizing target quality, regardless of firm 2's quality choice.

UNCERTAIN QUALITY INVESTMENT

We extend the uncertain quality choice model: to receive a distribution $\mathcal{U}[0, b_i]$, each firm has to incur an investment cost of αb_i^2. We find that α and B uniquely determines the class of pure strategy Nash equilibria in the quality stage (Proposition 6.10)

αB Values	Pure Strategy Nash Equilibrium (Equilibria)
$\alpha B \in (0, K^*)$	(B, B) (Dominant Strategy Equilibrium)
$\alpha B = K^*$	(B, B), $(0, B)$, and $(B, 0)$
$\alpha B \in (K^*, K^{**})$	$(0, B)$ and $(B, 0)$
$\alpha B \in [K^{**}, \infty]$	$(0, \tilde{b})$ and $(\tilde{b}, 0)$

Table 1. Pure Strategy Nash Equilibria Across αB Values

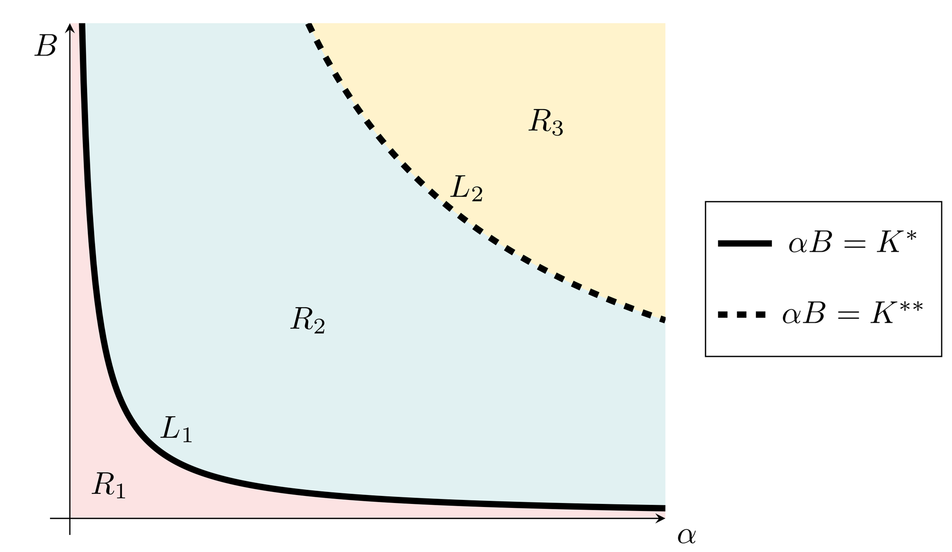

Figure 3. Equilibria Classes Across Regions in the (α, B)-State Space

Remarks: *Table 1* and *Figure 3* shows the regions of αB that yield different classes of Nash equilibria. As αB increases, firms will differentiate their quality distributions.

Proposition 6.10 implies that industries with cheap investment costs and low technological frontiers will experience more intense quality competition.

CONCLUSIONS

- Sufficient differences in returns to quality investment leads to a unique pure strategy Nash equilibrium in the quality stage.
- Stochastic quality realization is an *incentive* for quality investment.
  - Uncertain quality softens price competition and prevents firms from direct Bertrand competition.
  - Firms no longer need to intentionally differentiate their choices in the quality stage.

REFERENCES

Jean Tirole. *The Theory of Industrial Organization*. The MIT Press, 1988.

Paul Belleflamme and Martin Peitz. *Industrial Organization: Markets and Strategies*. Cambridge University Press, 2010.

I would like to thank **Professor Christiaan Hogendorn** for being an amazing advisor. His constant encouragement, creativity, and guidance helped make this project achieve its full potential. I also give many thanks to **Professor Gilbert Skillman** for his remarkable insights and suggestions. Professor Hogendorn and Professor Skillman both played an integral role in helping me shape this thesis from a (rather long) mathematical exercise into an in-depth analysis of an economic situation that readers will find interesting.